PS3 Q3

Let and be the eigenvalue and corresponding eigenvector of symmetric matrix .

1. The Jordan Normal Form

Since is a symmetric matrix, the eigenvalues are real and we can find a unitary matrix such that where is a diagonal matrix which entries are the eigenvalues of .

To build each column of is the orthonormal eigenvector of

1. Exponential matrix

Since we have , we have .

Recall

Hence,

1. The principal logarithm

Recall

Hence,

1. The principal fractional power

Recall for a nonzero complex scalar , we define where log is the principal logarithm. We can generalize this definition to matrices.

For a nonsingular matrix , we define

Since , we have

Following the idea of the exponential matrix and the principal logarithm, we obtain